B.Sc. Mathematics Syllabus

Techno India University, West Bengal

Semester	**Core Course (Credit - 6)**	**ABILITY ENHANCEMENT COMPULSARY COURSE – AECC (credit - 2)**	**SKILL ENHANCEMENT COURSE – SEC**	**DISCIPLINE SPECIFIC ELECTIVE – DSE (Credit - 6)**	**GENERIC ELECTIVE COURSE – GE (Credit - 6)**	**Total Credit**
Semester-I	Mathematics -I	Communicative English			GE-I	6+6+6+2=20
	Mathematics -II					
Semester-II	Mathematics -III	EVS			GE-II	6+6+6+2=20
	Mathematics -IV					
Semester-III	Algebra – III		SEC – I		GE-III	6+6+6+2=20
	Differential Equations					
Semester-IV	Riemann Integration & Series of Functions		SEC – II		GE-IV	6+6+6+2=20
	Numerical Analysis (Theory+Practical = 30+20)					
Semester-V	Algebra – IV			DSE-I		6+6+6+6=24
	Linear Programming & Probability			DSE-II		
Semester-VI	Metric Space & Complex Analysis			DSE-III		6+6+6+6=24
	Algebra – V			DSE-IV		
						Total=128

CHOICE OF DISCIPLINE SPECIFIC ELECTIVE - DSE

DSE – I	DSE – II
1. Industrial Mathematics 2. Combinatorics & Graph Theory	1. Portfolio Optimization 2. Theory of Equations
DSE - III	**DSE – IV**
1. Differential Geometry 2. General Mechanics	1. Number Theory 2. Hydrostatics

*More DSE papers to be announced later

Mathematics – I

Algebra – I (15)

- **Complex Numbers**: Polar representation of complex numbers, nth roots of unity, De Moivre's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number.
- **Theory of equations**: Fundamental Theorem of Classical Algebra(statement only) Relation between roots and coefficients, transformation of equation, Descartes rule of signs, solution of cubic and biquadratic equations: Cardan's and Ferrari's method.
- **Inequality**: The inequality involving AM≥ GM≥ HM, Cauchy-Schwartz inequality.
- **Matrix Algebra**: Properties of Determinant, linear independence, Systems of linear equations: row reduction and echelon forms, rank of a matrix, the matrix equation Ax=b, solution sets of linear systems, applications of linear systems.

Discrete Mathematics (10)

- Binary Relations. Equivalence relation and Partition: Their Equivalence
- Functions: Injective, Surjective, Bijective. Composition of functions, Invertible functions .
- Well-ordering property of positive integers, division algorithm, divisibility and Euclidean algorithm.
- Congruence relation between integers. Principles of Mathematical induction, Statement of Fundamental Theorem of Arithmetic.

Geometry (25)

- Pole and polars. Diameters and conjugate diameters. Systems of conics. Polar equation of a conic referred to a focus as pole. Equations of tangent, normal, chord of contact.
- **Sphere**: General Equation. Great circle, Sphere through the intersection of two spheres. Radical Plane, Tangent, Normal. Cone: Right circular cone. General homogeneous second degree equation. Section of cone by a plane as a conic and as a pair of lines. Condition for three perpendicular generators. Reciprocal cone. Cylinder: Generators parallel to either of the axes, general form of equation. Right-circular cylinder. Ellipsoid, Hyperboloid, Paraboloid: Canonical equations only. Tangent planes, Normal, Enveloping cone. Generating lines of hyperboloid of one sheet and hyperbolic paraboloid.

Mathematics – II

Differential Calculus – I (30)

- Rational numbers, Geometrical representations, Irrational number, Real number represented as point on a line — Linear Continuum. Basic properties of real number.
- Limits of functions (ε - δ approach), sequential criterion for limits, divergence criteria. Limit theorems, one sided limits. Infinite limits and limits at infinity.
- Continuous functions, sequential criterion for continuity and discontinuity. Algebra of continuous functions. Continuous functions on closed and bounded interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Classification of discontinuity, discontinuity of monotonic functions. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem on compact sets.
- Differentiability of a function at a point and in an interval, Caratheodory's theorem, algebra of differentiable functions. Relative extrema, interior extremum theorem.
- Statement of Rolle's Theorem and its geometrical interpretation. Mean value theorems of Lagrange and Cauchy. Statements of Taylor's and Maclaurin's Theorems with Lagrange's and Cauchy's from of remainders.
- Indeterminate Forms : L'Hospital's Rule : Statement and Problems only
- Successive derivative - Leibnitz's theorem and its application.
- Application of the principle of Maxima and Minima for a function of single variable in geometrical, physical and to other problems.

Sequence and Series (20)

- Sequences, bounded sequence, convergent sequence, limit of a sequence, lim inf, lim sup. Limit theorems. Monotone sequences, monotone convergence theorem. Subsequences, divergence criteria. Monotone subsequence theorem (statement only), Bolzano Weierstrass theorem for sequences. Cauchy sequence, Cauchy's convergence criterion.

- Infinite series, convergence and divergence of infinite series, Cauchy criterion, tests for convergence: comparison test, limit comparison test, D'Alembert's test, Raabe's test, Cauchy's nth root test, Gauss test, Logarithmic test, Integral test. Alternating series, Leibniz test. Absolute and conditional convergence, Rearrangement of series.

Mathematics – III

Differential Calculus – II (25)

- Functions of two and three variables : geometrical representations. Limit and Continuity for function of two variables. Partial derivatives. Chain Rule. Exact differentials.
- Functions of two variables - Successive partial Derivatives : Statement of Schwarz's Theorem on Commutative property of mixed derivatives. Euler's Theorem on homogeneous function of two and three variables.
- Applications of Differential Calculus: Concavity and inflection points, envelopes, asymptotes, curvature, curve tracing in cartesian coordinates.
- Maxima and minima of functions of not more than three variables Lagrange's Method of undetermined multiplier - Problems only.

Algebra – II (25)

- **Introduction to Group Theory** : Definition and examples taken from various branches (example from number system, roots of Unity, 2×2 real matrices, non singular real matrices of a fixed order). Elementary properties using definition of Group. Definition and examples of sub- group - Statement of necessary and sufficient condition and its applications.
- Definitions and examples of (i) Ring, (ii) Field, (iii) Sub-ring, (iv) Sub- field.
- **Vector spaces**, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces (with special emphesis on R^n over R).

Mathematics – IV

Integral Calculus (25)

- Evaluation of definite integrals.
- Integration as the limit of a sum (with equally spaced as well as unequal intervals).
- Reduction formulae and associated problems.
- Definition of Improper Integrals : Statements of (i) μ-test (ii) Comparison test (Limit from excluded) - Simple problems only.
- Use of Beta and Gamma functions (convergence and important relations being assumed).
- Double integration over rectangular region, double integration over non-rectangular region, Double integrals in polar co-ordinates, Triple integrals, triple integral over a parallelepiped and solid regions. Volume by triple integrals, cylindrical and spherical co-ordinates. Change of variables in double integrals and triple integrals.

Vector Calculus: (25)

- Triple product, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions.
- Definition of vector field, divergence and curl. Line integrals, applications of line integrals: mass and work. Fundamental theorem for line integrals, conservative vector fields, independence of path.
- Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stoke's theorem, The Divergence theorem.

Algebra – III (Linear Algebra)

- Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, Eigen values, Eigen vectors and characteristic equation of a matrix. Cayley-Hamilton theorem. Algebra of linear transformations.
- Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators. Eigen spaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator, canonical forms.
- Inner product spaces and norms, Gram-Schmidt orthogonalisation process, orthogonal complements, Bessel's inequality, the adjoint of a linear operator. Least squares approximation, minimal solutions to systems of linear equations. Normal and self-adjoint operators. Orthogonal projections and Spectral theorem.
- Application : Reflection properties of conics, rotation of axes and second degree equations, classification of conics using the discriminant.

Differential Equations

Ordinary Differential Equations (35)

- Order, degree and solution of an ordinary differential equation (ODE) in presence of arbitrary constants, Formation of ODE.
- First order equations : (i) Exact equations and those reducible to such equation. (ii) Euler's and Bernoulli's equations (Linear). (iii) Clairaut's Equations : General and Singular solutions.
- Second order linear equations : Second order linear differential equation with constant coefficients. Euler's Homogeneous equations.
- Second order linear differential equation with variable coefficients: Cauchy – Euler Equation.
- Second order differential equation : (i) Method of variation of parameters (ii) Method of undetermined coefficients.
- Orthogonal Trajectories
- Simultaneous differential equations, Simple eigen-value problem.
- Lipschitz condition and Picard's Theorem (Statement only)

Partial Differential Equations (15)

- Partial differential equations – Basic concepts and definitions.
- Formation of PDE, Mathematical problems.
- First order equations: classification, construction and geometrical interpretation.
- Lagrange's and Charpit's method for solving PDE.
- Method of characteristics for obtaining general solution of quasi linear equations. Canonical forms of first-order linear equations.
- Method of separation of variables for solving first order partial differential equations.

Riemann Integration & Series of Functions

- Riemann integration: inequalities of upper and lower sums, Darboux integration, Darboux theorem, Riemann conditions of integrability, Riemann sum and definition of Riemann integral through Riemann sums, equivalence of two definitions. Riemann integrability of monotone and continuous functions, properties of the Riemann integral; Integrability of functions with infinitely many discontinuities having finitely many limit points, Definition and integrability of piecewise continuous and monotone functions. Intermediate Value theorem for Integrals; Fundamental theorem of Integral Calculus.
- Pointwise and uniform convergence of sequence of functions. Theorems on continuity, derivability and integrability of the limit function of a sequence of functions. Series of functions; Theorems on the continuity and derivability of the sum function of a series of functions; Cauchy criterion for uniform convergence and Weierstrass M-Test, Weierstrass approximation theorem.
- Fourier series: Definition of Fourier coefficients and series, Reimann Lebesgue lemma, Bessel's inequality, Parseval's identity, Dirichlet's condition. Examples of Fourier expansions and summation results for series.
- Power series, radius of convergence, Cauchy Hadamard theorem. Differentiation and integration of power series, Abel's theorem, Dirichlet's Theorem.

Numerical Analysis (Theory + Practical = 30 + 20)

Numerical Theory

- Algorithms. Convergence. Errors: relative, absolute. Round off. Truncation.
- Interpolation: Lagrange and Newton's methods. Error bounds. Finite difference operators. Gregory forward and backward difference interpolation. Numerical differentiation: Methods based on interpolations, methods based on finite differences.
- Transcendental and polynomial equations: Bisection method, Newton's method, secant method, Regula-falsi method, fixed point iteration, Newton-Raphson method. Rate of convergence of these methods.
- System of linear algebraic equations: Gaussian elimination and Gauss Jordan methods. Gauss Jacobi method, Gauss Seidel method and their convergence analysis.
- Numerical Integration: Newton Cotes formula, Trapezoidal rule, Simpson's 1/3rd rule, Simpsons 3/8th rule, Weddle's rule, Boole's Rule. midpoint rule, Composite trapezoidal rule, composite Simpson's 1/3rd rule, Gauss quadrature formula.
- Ordinary differential equations: The method of successive approximations, Euler's method, the modified Euler method, Runge-Kutta methods of orders two and four.

Numerical Practical using C/C++

- Forward and Backward Difference table
- Newton's Forward Interpolation Method, Newton's Backward Interpolation Method, Lagrange's Interpolation Method
- Numerical Integration by Trapezoidal rule and Simpson's 1/3rd rule
- Numerical Solution of Algebraic and Transcendental Equations by Bisection Method, Regula Falsi Method and Newton Raphson Method
- Numerical Solution of a System of Equations by Gauss Elimination Method, Gauss Jordon Method, Jacobi Iteration Method and Gauss Seidel Method
- Numerical Solution of Initial value problems by Euler's Method, Modified Euler's Method and 4 th order Runge Kutta Method

Algebra – IV (Abstract Algebra)

- Symmetries of a square, dihedral groups, permutation groups and quaternion groups (through matrices)
- Examples of subgroups, centralizer, normalizer, center of a group, product of two subgroups.
- Properties of cyclic groups, classification of subgroups of cyclic groups. Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.
- External direct product of a finite number of groups, normal subgroups, factor groups, Cauchy's theorem for finite abelian groups.
- Group homomorphisms, properties of homomorphisms, Cayley's theorem, properties of isomorphisms. First, Second and Third isomorphism theorems.
- Properties of rings, subrings, integral domains and fields, characteristic of a ring. Ideal, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals.
- Ring homomorphisms, properties of ring homomorphisms. Isomorphism theorems I, II and III, field of quotients.

Linear Programming and Probability

Linear Programming (25)

- Motivation of Linear Programming problem. Statement of L.P.P. Formulation of L.P.P. Slack and Surplus variables. L.P.P. is matrix form. Convex set, Hyperplane, Extreme points, convex Polyhedron, Basic solutions and Basic Feasible Solutions (B.F.S.). Degenerate and Non-degenerate B.F.S.
- The set of all feasible solutions of an L.P.P. is a convex set. The objective function of an L.P.P. assumes its optimal value at an extreme print of the convex set of feasible solutions, A.B.F.S. to an L.P.P. corresponds to an extreme point of the convex set of feasible solutions.
- Fundamental Theorem of L.P.P. (Statement only) Reduction of a feasible solution to a B.F.S. Standard form of an L.P.P. Solution by graphical method (for two variables), by simplex method and method of penalty. Concept of Duality. Duality Theory. The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Dual problems with at most one unrestricted variable, one constraint of equality. Transportation and Assignment problem and their optimal solutions.

Probability (25)

- Elements of probability Theory: Random experiment, Outcome, Event, Mutually Exclusive Events, equally likely and Exhaustive. Classical definition of probability, Theorems of Total Probability, Conditional probability and Statistical Independence. Bayes' Theorem. Problems, Shortcoming of the classical definition. Axiomatic approach problems, Random Variable and its Expectation, Theorems on mathematical expectation. Joint distribution of two random variables.
- Theoretical Probability Distribution Discrete and Continuous (p.m.f., p.d.f.) Binomial, Poisson and Normal distributions and their properties.

Metric Space & Complex Analysis

Metric spaces (25)

- Metric spaces: Definition and examples. Open and closed balls, neighbourhood, open set, interior of a set. Limit point of a set, closed set, diameter of a set, subspaces, dense sets, separable spaces. Sequences in metric spaces, Cauchy sequences. Complete metric spaces, Cantor's theorem.
- Continuous mappings, sequential criterion and other characterizations of continuity. Uniform continuity. Connectedness, connected subsets of R. Compactness: Sequential compactness, Heine-Borel property, totally bounded spaces, finite intersection property, and continuous functions on compact sets. Homeomorphism. Contraction mappings. Banach fixed point theorem and its application to ordinary differential equation.

Complex Analysis (25)

- Limits, limits involving the point at infinity, continuity. Properties of complex numbers, regions in the complex plane, Stereographic projections, functions of complex variable. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability, Harmonic functions. Analytic functions, examples of analytic functions, exponential function, logarithmic function, trigonometric function, derivatives of functions.
- Definite integrals of functions. Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals. Cauchy- Goursat theorem, Cauchy integral formula. Liouville's theorem and the fundamental theorem of algebra. Convergence of sequences and series, Taylor series and its examples. Laurent series and its examples, absolute and uniform convergence of power series

Algebra – V

- Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups, Characteristic subgroups, Commutator subgroup and its properties.
- Properties of external direct products, the group of units modulo n as an external direct product, internal direct products, Fundamental theorem of finite abelian groups.
- Group actions, stabilizers and kernels, permutation representation associated with a given group action. Applications of group actions. Generalized Cayley's theorem. Index theorem.
- Groups acting on themselves by conjugation, class equation and consequences, conjugacy in Sn, p groups, Sylow's theorems and consequences, Cauchy's theorem, Simplicity of An for $n \geq 5$, nonsimplicity tests.
- Polynomial rings over commutative rings, division algorithm and consequences, principal ideal domains, Euclidean domains.
- Factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, and unique factorization in Z [x]. Divisibility in integral domains, irreducible, primes unique factorization domains.

NOTE: SEC I, II and GEC I, II, III, IV not included since it may vary from session to session.