

### 2-Year Master of Science (M.Sc.) Curriculum and Syllabus for Mathematics

# **Fourth Semester**

Course Code	Course Title	Contact Hrs/Week			Credit
		L	Т	Р	Credit
Theory					
TIU-PMA-T208	PARTIAL DIFFERENTIAL EQUATIONS	3	1	0	4
TIU-PMA-T210	DISCRETE MATHEMATICS	3	1	0	4
TIU-PMA-T2**	ELECTIVE II	3	1	0	4
TIU-PMA-P298	PROJECT & VIVA VOCE	0	0	0	5
TIU-PMA-G298	GRAND VIVA VOCE	0	0	0	3
Practical					
TIU-PMA-L298	CAREER ADVANCEMENT & SKILL DEVELOPMENT – IV	3	0	0	3
Sessional					
TIU-PES-S298	ENTREPRENEURSHIP SKILL DEVELOPMENT	0	0	0	2
	Total Credits				25

## PARTIAL DIFFERENTIAL EQUATIONS

Introduction, Cauchy-Kowalewski's theorem (statement only) classification of second order PDE. Reduction of linear and quasilinear equations in two independent variables to their canonical forms, characteristic curves. Well-posed and ill-posed problems. Hyperbolic Equations: The vibration of a string. Formulation of mixed initial and boundary value problem. Existence, uniqueness and continuous dependence of the solution to the initial conditions. D'Alembert's formula for the vibration of an infinite string. Method of separation of variables. Investigation of the conditions under which the infinite series solution convergence and represents the solution. Riemann method of solution, Problems. Rectangular and circular membranes problems. Elliptic equations: Occurrence of Laplace's equation. Fundamental solutions of laplace's equation in two independent variables. Laplace equation in polar, spherical polar and in cylindrical polar coordinates, Minimum – Maximum theorem and its consequences. Boundary value problems, Dirichlets and Neumann's interior and exterior problems, uniqueness and continuous



dependence of the solution on the boundary conditions. Method for the solution of Laplace's equations in two dimensions interior and exterior Dirichlet's problem for a circle, and a semi circle, Green's function for the Laplace equation in two dimensions. Parabolic equation: Conduction of heat in a bounded strip, First boundary value problem, Maximum-Minimum theorem and its consequences, uniqueness, continuous dependence of the solution and existence of the solution.

#### Books:

- 1. Elements of Partial Differential Equations by IN Sneddon
- 2. Advanced Differential Equations by MD Raisinghania

## **DISCRETE MATHEMATICS**

### The Foundations: Logic and Proofs

Propositional Logic, Propositional Equivalences, Predicates and Quantifiers, Nested Quantifiers, Rules of Inference, Introduction to Proofs, Proof methods and strategy.

### **Introduction to Combinatorics**

The Mathematics of Choice : The fundamental counting principle, Pascal's triangle, Error-correcting codes, Combinatorial identities, Four ways to choose, The binomial and multinomial theorems, Partitions, Elementary symmetric functions.

Recurrence: Some examples, The auxiliary equation method, Generating functions, Derangements, Catalan numbers.

Vertex Colorings of Graphs, Edge Colorings of Graphs.

The Inclusion-Exclusion Principle: The principle, Counting surjections, Counting labeled trees, Scrabble, The Menage problem.

Latin squares and Hall's theorem: Latin squares and orthogonality, Magic squares, Systems of distinct representatives, From Latin squares to Affine planes.

Schedules and 1-Factorizations: The circle method, Bipartite tournaments and 1-factorizations of  $K_{\Pi,\Pi}$ , Tournaments from orthogonal Latin squares.

Introduction to designs: Balanced incomplete block designs, Resolvable designs, Finite projective planes, Hadamard matrices and designs, Difference methods, Hadamard matrices and codes.



# CASD-IV

Preparation of MSc dissertation and project presentation.

## **ELECTIVE II**

### ADVANCED OPERATIONS RESEARCH

### Job Sequencing

Sequencing problems, Solution of sequencing problems, Processing n jobs through two machines, Processing n jobs through three machines, Optimal solutions, Processing of two jobs through m machines, Graphical method, Processing n jobs through m machines.

### **Project Scheduling and Network Analysis**

PERT and CPM review, Crashing of an activity, Crash-cost slope, Time-cost trade, Solution of network problems using Simplex technique. Time estimates for PERT, Probability of completion of a project within a scheduled time.

#### **Replacement Models**

Replacement problem, Types of replacement problems, Replacement of capital equipment that varies with time, Replacement policy for items where maintenance cost increases with time and money value is not considered, Money value, Present worth factor (pwf), Discount rate, Replacement policy for item whose maintenance cost increases with time and money value changes at a constant rate, Choice of best machine, Replacement of low cost items, Individual replacement policy, Mortality theorem, Group replacement policy.

### Inventory and Queuing Models

Inventory models: EOQ and EPQ models and their applications, Basic review systems and single period model and their applications; queuing models: M/M/1 Queues and applications, M/M/c and M/M/c/k Queues and their applications



# FUZZY SET THEORY

Basic concepts of fuzzy sets, fuzzy logic, operations on fuzzy sets, fuzzy relations, equivalence and similarity relations, ordering, morphisms, fuzzy relation equations, fuzzy measures, probability measures, possibility and necessity measures, measures of uncertainty, dissonance, confusion and nonspecificity. Principles of uncertainty and information. Applications of fuzzy sets in management, decision making, computer science and systems science.

# **MECHANICS OF CONTINUA**

Principles of continuum mechanics, axioms. Forces in a continuum. The idea of internal stress. Stress tensor. Equations of equilibrium. Symmetry of stress tensor. Stress transformation laws. Principal stresses and principal axes of stresses. Stress invariants. Stress quadric of Cauchy. Shearing stresses. Mohr's stress circles. Deformation. Strain tensor. Finite strain components in rectangular cartesion coordinates. Infinitesimal strain components. Geometrical interpretation of infinitesimal strain components. Principal strain and principal axes of strain. Strain invariants. The compatiability conditions. Compatibility of strain components in three dimensions. Constitutive equations. Inviscid fluid. Circulation. Kelvins energy theorem. Constitutive equation for elastic material and viscous fluid. Navier and Stokes equations of motion. Motion of deformable bodies. Lagrangian and Eulerian approaches to the study of motion of continua. Material derivative of a volume integral. Equation of continuity. Equations of motion. Equation of angular momentum. Equation of Energy. Strain energy density function.

## ADVANCED ALGEBRA - II

Multilinear Algebra



Determinants, Tensor Algebras, Symmetric Algebras, Exterior Algebras, Homomorphisms of Tensor Algebras, Symmetric and Alternating Tensors. Structure of Rings

Artinian rings, Simple rings, Primitive rings, Jacobson density theorem, Wedderburn - Artin theorem on simple (left)Artinian rings. The Jacobson radical. Jacobson semisimple rings, subdirect product of rings, Jacobson semisimple rings as subdirect products of primitive rings, Wedderburn - Artin theorem on Jacobson semisimple (left)Artinian rings. Simple and Semisimple modules, Semisimple rings, Equivalence of semisimple rings with Jacobson semisimple (left)Artinian rings. Properties of semisimple rings. Characterizations of semisimple rings in terms of modules.

#### **Group Representations**

Representations, Group-Rings, Maschke's Theorem, Character of a Representation, Regular Representations, Orthogonality Relations, Burnside Two-Prime Theorem.

## **FLUID MECHANICS – II**

Six governing equations of fluid motion, crocco-vazsonyl equation. Propagation of small disturbances in a gas. mach number. Dynamics similarity of two flows. Circulation theorem. Permanence of irrotational motion. Bernoulli's integral for steady isentropic and irrotational motion. Polytropic gas. Critical speed. Equation satisfied velocity potential and stream functions. Prandtl-Mayer fluid past a convex corner. Steady flow through a De Level nozzle. Normal and oblique shock wave shock polar diagram one dimensional similarity flow. Steady linearised subsonic and supersonic flows. Prandtl-Glauert transformation. Flow along a wavy boundary flow past a slight corner. Jangen-rayleigh method of approximation. Thin supersonic wind Ackeret's formula. Legendre and molenbroek transformations Chaplygin's equation for stream function. Solution of chaplygin's equation. Subsonic gas jet problem limiting line. Motion due to a two dimensional source and a vortex Karman-Tsien approximation. Two



dimensional steady flow : Riemann invarience. Method of characteristic. Transonic flow. Law transonic similarity. Euler's-Tricomi equation and its fundamental solution. Hypersonic flow.

## BANACH ALGEBRA-II

Commutative-\*-algebras, Self-dual vector spaces and \*-representation, positive functionals and \*-representations on Hilbert space, General properties of B\*-algebras, structure of ideals and representations of B\*-algebras.

Algebras of operators : Elements of algebras of compact operators. C\*algebra, W\*-algebra, positive elements and positive linear functionals on C\*algebra, weak topology and various topologies on W\*-algebra, ideals in W\*algebra, spectral resolution of self- adjoint elements in a W\*-algebra.

## ASTROPHYSICS - II

Plasma, black Body, Cherenkov & Synchroton Radiation, Accreation as source of radiation, Quasar as source of radiation, Compton effect effect, Bremsstrallung Radiation.

Formation of Galactic Structure - different Theories : - Formation of our Galaxy. Formation of Galaxy in Evolutionary Universe. Formation of Galaxy in Steady State Universe. Possibility of galactic structure formation through Explosion.

Hubble's Law & Expansion of Universe - Big Bang Model, Uniformity of Large Scale Structure of the Universe, Origin of Cosmic Rays, Origin of Galaxies and the Universe.



### **GENERAL THEORY OF RELATIVITY AND COSMOLOGY-II**

What is cosmology? Homogeneity and isotropy of the universe. The Weyl cosmological principle. General relativistic cosmological Postulate. The Cosmological Olbers Paradox. The models. observations. The Friedman Cosmological Models (dust and radiation models). Cosmologies with a non-zero  $\lambda$ . Hubble's Law, the age of the4 Universe. Gravitational red shift and Cosmological redshift. The spherically symmetric space-time : Schwarzschild solution Patrick orbits in the Schwarzschild space-time. Newtonian approximation. Photon orbits. Birkhoffs theorem. Equilibriumof Massive spherical objects. The Schwarzschild Interior solution. The interior structure of the star. Realistic stars and gravitational collapse. White dwarfs, Neutron stars. Gravitational collapse of a homogeneous dust ball. Schwarzschild black hole. Simple idea of black hole physics.

# RINGS OF CONTINUOUS FUNCTIONS-II

 $\beta$ X for various spaces X and their cardinalities; Gelfand-Kolmogoroff Theorem; Realcompact spaces and Hewitt real compactification  $\cup$ X of X and their relations with  $\beta$ X, Zero-dimensional spaces, Extremally disconnected spaces, Basically disconnected spaces, P spaces, F-spaces X and their characterizations in terms of C(X). Some interesting Problems.

# MATHEMATICAL STATISTICS - II

Bayesian inference. Loss function. Decision function. Complete and minimal complete classes. Admissibility. Minimax solution. Bayes solution. Inference related to linear models - one and several parametric functions. Setup with restrictions. Analysis of variance. One-way classified data. Two-way classified data with single/multiple observations per cell. General theory of regression. Test for an assigned regression function. Principal component analysis. Basic concepts of factor analysis. Non-parametric inference. Distribution of order statistics. Robustness. Distribution-free methods.



Standard non-parametric tests for location and independence. Sequential probability ratio test and its properties. Fundamental identity. Sequential estimation.

## PROBABILITY AND STOCHASTIC PROCESS - II

Markov chain with stationary transition probabilities. Communication. Classification of states. Periodicity. Transient, null recurrent and positive recurrent states. Limiting probabilities. Ergodicity. Algebraic theory.

Continuous-parameter markov chains. Simple Markov processes. Poisson process Characteristic functional. Polya process. Birth-and-death process.

Branching process. Galton-Watson process. Continuous-parameter branching process. Age-dependent branching process.

General theory of continuous-time processes. Kolmogoroff's forward and backward equations. Fokker-Planck equations. Wiener process.

## **TOPOLOGICAL VECTOR SPACES – II**

Locally convex spaces, Spaces:  $\ell_p$ , Lp, C(p) and  $\ell_\infty$  Perfect, simple and symmetric spaces, duality:  $\ell_1$ ,  $\ell_\infty$ ,  $\ell_p$ ,  $\ell_q$ , C<sub>0</sub>, Lp. Matrix transformations between C and C<sub>0</sub>,  $\ell_1$  and  $\ell_\infty$ ,  $\ell_1$  and  $\ell_p$ ,  $\ell_p$  and  $\ell_q$ . Banach limits, almost convergence, absolute convergence, matrix transformations between almost convergence and absolutely convergent sequences. Banach-Alaoglu theorem, Krein-Milman theorem, convexity, Distribution theory

### **OPERATOR THEORY**

Bounded linear Operators, Dual space considerations, Representation of duals of the spaces c\_00 with p-norms, l\_p, C[a.b] and c with supremum - norm, Reflexivity, Weak and weak\* convergences. Self-adjoint, Normal and unitary operators



Compact linear operators, Spectral properties of compact linear operators on a normed linear space. Fredholm alternative theorem, Fredholm alternative for integral equations. Spectral theory: Resolvent set, Spectrum, Point spectrum, Continuous spectrum, Residual spectrum, Approximate point spectrum, Spectral radius, Specrtral properties of a bounded linear operator, Spectral mapping theorem for polynomials.

Numerical range, Numerical radius, Convexity of numerical range, Closure of numerical range contains the spectrum

## DYNAMICAL SYSTEMS

Autonomous system of differential equations. Existence and uniqueness of solutions (statement only)., Flows, phase space, orbits in phase space, equilibrium points and their nature, saddle point, node, focus points, stable , unstable and center subspaces. Definition of stability, Lyapunov function. Hartman- Grobman theorem (statement only). Poincare map, periodic orbits, invariant sets, limit points and limit cycles, Poincare index, attracting and repelling sets, trapping regions, Poincare Benedixson theorem (statement only). Bifurcations – Saddle-node, Transcritical, Pitchfork and Hopf bifurcations.

# PROJECT AND VIVA VOCE M.Sc. Project

Each student shall submit three proposals for the project, he/she wants to undertake in order of preference from which the final topic may be selected. The project has to showcase the analytical skill and/or numerical skill of the students.

### M.Sc. Project and Project Viva

On completion of the project, each student has to present his/her work to a Scrutiny Committee. It is mandatory for the students to prepare the



presentation using any Latex-based presentation software. The students will be required to appear before a minimum of one External Expert. The Expert/s may be invited from any institute/university of repute.

## **GRAND VIVA VOCE**

The students will be required to appear before a minimum of one External Expert. The Expert/s may be invited from any institute/university of repute.