## TECHNO INDIA UNIVERSITY <br> W ESTBENGAL

EM 4, Sector V, Salt Lake, Kolkata-700091, West Bengal, India Phone: +91 9836544416/17/18/19, Fax: +91 3323571097

## 2-Year Master of Science (M.Sc.) Curriculum and Syllabus for Mathematics

Third Semester

| Course Code | Course Title | Contact Hrs/Week |  |  | Credit |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | L | T | P |  |
| Theory |  |  |  |  |  |
| TIU-PMA-T201 | ALGEBRA - II | 3 | 0 | 0 | 4 |
| TIU-PMA-T205 | OPTIMIZATION TECHNIQUES | 3 | 1 | 0 | 4 |
| TIU-PMA-T207 | GRAPH THEORY | 3 | 0 | 0 | 4 |
| TIU-PMA-T209 | INTEGRAL EQUATION AND VARIATIONAL METHODS | 3 | 1 | 0 | 4 |
| TIU-PMA-T2** | ELECTIVE - I | 3 | 1 | 0 | 4 |
| Practical |  |  |  |  |  |
| TIU-PMA-L297 | CAREER ADVANCEMENT \& SKILL DEVELOPMENT - III | 0 | 0 | 4 | 3 |
| Sessional |  |  |  |  |  |
| TIU-PES-S299 | ENTREPRENEURSHIP SKILL DEVELOPMENT | 0 | 0 | 0 | 2 |
|  | Total Credits |  |  |  | 25 |

## ALGEBRA-II

Field Extensions: Algebraic and Transcendental Extensions, Finite Extension, Algebraic Closure of a field, Algebraically Closed Field, Splitting Field of a polynomial, Multiple roots, Normal Extension, Separable Extension, Impossibility of some constructions by straightedge and compass. Finite Fields and their properties, Galois Group of automorphisms and Galois Theory.

## Books:

1. Abstract Algebra by DS Dummit and RM Foote
2. Fundamentals of Abstract Algebra by Malik, Mordersen and Sen

## 3. Topics in Abstract Algebra by I.N. Herstein

## OPTIMIZATION TECHNIQUES

Non-linear programming: Lagrangian function, NLPP with equality constraint, NLPP with inequality constraint, Kuhn-Tucker (KT) conditions, Quadratic programming, Convex Programming, Separable Programming. Integer Programming: Branch and bound algorithm, Cutting plane methods for pure and mixed Integer programming problems, Knap-sack problem, travelling salesman problem, Dynamic Programming: Bellman's principle of optimality and recursive relationship of dynamic programming for various optimization problems. Stochastic Programming: Stochastic programming with one objective function. Stochastic linear programming. Two stage programming technique. Chance constrained programming technique, Genetic Algorithms, Ant Colony Optimization, Particle Swarm Optimization.

## Books:

1. Optimization for Engineering Design: Algorithms and Examples by K Deb
2. An algorithm to Genetic Algorithm by Melanie Mitchell

## GRAPH THEORY

Introduction to Graphs : The concept of a graph, Paths in graphs, Graphs and graph models, Graph terminology and special types of graphs, Bipartite graphs, Complete graphs, External graphs, Intersection graphs, Operations on graph, Graph Isomorphism. Blocks : Cutpoints, bridges and blocks. Block graphs and cutpoint graphs. Trees : Introduction to trees and characterizations, Applications of Trees, Spanning Trees, Minimum Spanning Trees, Trees in computer science, Centers and centroids, Block-cutpoint trees, Independent cycles and cocycles, Matroids. Connectivity: Connectivity and line-connectivity, Graphical version of Menger's theorem. Traversability: Eulerian Graphs, Hamiltonian Graphs. Coverings and Matching: Coverings and independence, Critical points and lines, Matching, Maximum Matching Problem, Minimum covering problems. Representing Graphs: Adjacency matrix, Incidence matrix, Cycle matrix. Planarity: Plane and planar graphs, Outerplanar graphs, Kuratowski's theorem, other characterizations of planar graphs. Colorability: Vertex coloring, Chromatic number, Edge coloring, Five color theorem, Unique 2 colourable graphs. Directed Graphs: Basic definitions, Type of Connectedness, Covers and Bases, Distance concepts and matrices, Connectivity,

Acyclic digraphs, Cycles and traversability, Orientations and Tournaments. Network Flows: Max Flow - Min Cut Theorem, Menger's Theorem.

## Books:

1. N. Deo ; Graph Theory with Application to Engineering and Computer Science;
2. John Clark and Derek Allan Holton ; A First Look at Graph Theory;
3. F. Harary ; Graph Theory;

## INTEGRAL EQUATION AND VARIATIONAL METHODS

Integral Equations: Basic concepts, Volterra integral equations, relationship between linear differential equations and Volterra equations, resolvent kernel, method of successive approximations, convolution type equations, Volterra equation of the first kind. Abel's integral equation. Fredholm integral equations, Fredholm equations of the second kind, the method of Fredholm determinants, iterated kernels, integral equations with degenerate kernels, eigen values and eigen functions of a Fredholm alternative, construction of Green s function for BVP, singular integral equations. Existence and uniqueness of continuous solutions of Fredholm and Volterra's integral equation of second kind.

Calculus of Variations: Euler - Lagrange equations, degenerate Euler equations, Natural boundary conditions, transversality conditions, simple applications of variational principle, sufficient conditions for extremum. Variational formulation of BVP, minimum of quadratic functional. Approximate methods - Galerkin's method, weighted-residual methods, RayleighRitz method. Variational methods for time dependent problems.

## Books:

1. Calculus of Variations with Applications by AS Gupta
2. Integral Equations by FG Tricomi
3. Integral Equations and Boundary Value Problems by MD Raisinghania

| S.NO. | CONTENT |
| :--- | :--- |
| 1 | Installation of the software LaTeX |
| 2 | Understanding Latex compilation <br> Basic Syntex, Writing equations, Matrix, Tables |
| 3 | Page Layout - Titles, Abstract Chapters, Sections, Referrences, <br> Equation references, citation. <br> List making environments <br> Table of contents, Generating new commands, Figure handling <br> numbering, List of figures, List of tables, Generating index. |
| 4 | Packages: Geometry, Hyperref, amsmath, amssymb, algorithms, <br> algorithmic graphic, color, tilez listing. |
| 5 | Classes: article, book, report, beamer, slides. IEEtran. |
| 6 | Applications to: <br> Writing Resumae <br> Writing question paper <br> Writing articles/ research papers <br> Presentation using beamer. |

Solving NET/GATE question papers.

## ELECTIVE - I

## ADVANCED ALGEBRA - I

## Modules Theory

Left and Right Modules over a ring with identity, Cyclic Modules, Free Modules, Fundamental Structure Theorem for finitely generated modules over a PID and its applications to finitely generated abelian groups.

Modules and Module Homomorphisms, Submodules and Quotient Modules, Operations on submodules, Direct Sum and Product, Finitely Generated Modules, Free Modules. Tensor Products of modules, Universal Property of the tensor product, Restriction and Extension of Scalars, Algebras. Exact Sequences, Projective and Injective Modules, Five Lemma, Projective Modules and $\operatorname{Hom}_{\mathrm{R}}(M,-)$, injective modules and $\operatorname{Hom}_{\mathrm{R}}(-, M)$.

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## Commutative Ring Theory

Rings and Ring Homomorphisms, Ideals, Quotient Rings, Zero-divisors, Nilpotent elements, Units, Prime and Maximal ideals, Nil-radical and Jacobson radical, Nakayama's Lemma, Operations on Ideals, Prime Avoidance, Chinese Remainder Theorem, Extension and Contraction of ideals. Rings and Modules of Fractions, Local Properties, Extended and contracted ideals in rings of fractions. Noetherian Rings, Primary Decomposition in Noetherian Rings.

## FLUID MECHANICS - I

Lagrange's and Euler's methods in fluid motion. Equation of continuity, Boundary conditions and boundary surface stream lines and paths of particles. Irrotational and rotational flows, velocity potential. Bernoulli"s equation. Impulsive action equations of motion and equation of continuity in orthogonal curvilinear co-ordinate. Euler's equation of motion.

Theory of irrotational motion flow and circulation. Permanence irrotational motion. Connectivity of regions of space. Cyclic constant and acyclic and cyclic motion. Kinetic energy. Kelvin's minimum. Energy theorem. Uniqueness theorem. Dimensional irrotational motion.

Function. Complex potential, sources. sinks, doublets and their images circle theorem. Theorem of Blasius. Motion of circular and elliptic cylinders. Circulation about circular and elliptic cylinder. Steady streaming with circulation. Rotation of elliptic cylinder. Theorem of Kutta and Juokowski. Conformal transformation. Juokowski transformation. Schwartz-chirstoffel theorem.

Motion of a sphere. Stoke's stream function. Source, sinks, doublets and their images with regards to a plane and sphere.

Vortex motion. Vortex line and filament equation of surface formed by stream lines and vortex lines in case of steady motion. Strength of a filament. Velocity field and kinetic energy of a vortex system. Uniqueness theorem rectilinear vortices. Vortex pair. Vortex doublet. Images of a vortex with regards to plane and a circular cylinder. Angle infinite row of vortices. Karman's vortex sheet

Waves: Surface waves. Paths of particles. Energy of waves. Group velocity. Energy of a long wave.

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## BANACH ALGEBRA-I

General idea of Banach Algebras. Definitions and some examples. Regular and singular elements. Topological divisors of zero. The Spectrum. The formula for the Spectral radius.

The radical, semi-simplicity, ideals, maximal ideals space, structure of semisimple Banach algebras.

The carrier space and the Gelfand representation theorem, algebras of functions, The Silov boundary, representation of the carrier space, homomorphisms of certain function algebras into a Banach algebra, direct-sum decomposition and related results.

Involution in Banach algebras, the Gelfand-Neumark theorem.

## ASTROPHYSICS-I

Application of General Relativity to Astrophysics

1. Compact Objects, White dwarfs, Neutron stars and Black holes. Brief history of the formation and evolution of stars.
2. Schwarzschild exterior solution, Birkhoff's theorem, Schwarzschild singularity, Kruskal transformation, Schwarzschild Black hole. Motion of test particles around Schwarzschild black hole. Kerr metric and Kerr black holes (without deduction of solution). Horizons of Schwarzschild and Kerr black holes. Laws of black hole thermodynamics (statements only).
3. Interior of Schwarzschild metric, massive objects, Openheimer - Volkoff limit, Gravitational lensing, Quasars, Pulsars, Supernova.
4. Openheimer-Snydder non static dust model, Gravitational collapse.
5. Accretion into compact objects, Boltzmann formula, Saha Ionization equation, H-R diagram

## GENERAL THEORY OF RELATIVITY AND COSMOLOGY-I

Minkowski spacet-time : Past and future Cauchy development, Cauchy surface. DeSitter and anti-de Sitter space-times. Robertson-Walker spaces. Spatially homogeneous space-time models. The Schwarzschild and Reissner - Nordstrom solutions. Kruskal diagram. Causal structure. Orientability. Causal curves. Causality conditions. Cauchy developments. Global hyperbolicity. The existence of Geodesics. The Causal boundary of space-time. Asymptotically simple spaces.

## RINGS OF CONTINUOUS FUNCTIONS-I

Rings $C(X)$ and $C_{-}(X)$ for a topological space $X$, Zero sets $Z(f)$ and their properties, C-embedded and C*-embedded subsets of X; Uryshon' s Extension Theorem, z-ideals of $C(X)$ and z-filters on $X$ and their relations. Tychonoff spaces and M. H. Stone Theorem, Structure space of C(X) and Banach-Stone Theorem, Stone-Cech compactification $\beta X$ of a Tychonoff space $X$.

## MATHEMATICAL STATISTICS - I

Review: Population, Sample, Parameter, Statistic, Sampling Distribution. Estimation of parameters. Unbiasedness. Consistency. Sufficiency. Cramer-Rao bound. Rao-Blackwell theorem. Efficiency. Maximum likelihood estimators. Properties. Case of several parameters.
Tests of statistical hypotheses. Simple and composite hypothesis. Size and power of a test. NeymanPearson lemma and UMP test. Unbiasedness and similarity. UMPU and UMPS tests. Gauss-Markoff setup and least squares estimation. LS estimation with restriction on parameters. Simultaneous estimation of parametric functions. Multivariate normal distribution. Hotelluing $\mathrm{T}^{2}$.

## PROBABILITY AND STOCHASTIC PROCESS - I

Fields and $\sigma$-fields. Probability as a measure. Random variables. Probability distribution. Expectation. Moments. Absolute continuity and density function. Characteristic function. Inversion theorem. Convergence in probability. Weak convergence and continuity theorem for characteristic functions. Weak and strong law of large numbers. Law of the iterated logarithm. Lindeberg-Levy central limit theorem. Central limit theorem with Lindeberg and Liapunoff conditions. Definition and

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classification of stochastic processes. Simple random walk and gambler's ruin problem. Probability of ultimate ruin. Expected duration of game.

## TOPOLOGICAL VECTOR SPACES - I

Topological vector spaces, linear maps, types of topological vector spaces locally convex, locally bounded, locally compact spaces, metrizable and linear metric spaces, F spaces, Frechet spaces, FH, FK spaces, paranorm, separation properties, linear mapping, bounded linear transformation, seminorm and locally convexity, Quotient spaces, Baires theorem, Banach Steinhaus theorem, open mapping, closed graph theorem, Hahn-Banach theorem.

